

Strut-and-tie model for high strength concrete beams with combination of links and horizontal web bar

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Abstract

High Strength Concrete (HSC) is an aesthetic and durable material for bridge and viaduct construction. One of HSC advantages is that columns will have smaller cross section hence their visual obstruction to their surrounding environment will be reduced. Horizontal Web Bar (HWB), figure 2, has a number of advantages such as improved shear capacity of RC beams. This paper discusses shear behaviour of HSC beams with HWB and recommends a Strut-and-tie model (STM) for this structural system, figure 6.

A number of HSC and Normal Strength Concrete (NSC) beams were tested in order to compare their shear resistance. Furthermore an equal numbers of HSC and NSC beams with HWB were tested to failure, table 1.

The rules for estimating the contribution of HWB to the shear resistance were investigated by using the experimental measurement of strains in the steel as well as other available tests results.

Finite Element analysis was performed on HSC beams with HWB. The acquired numerical results were compared with those of experimental strains obtained by strain gauges of stirrups, tension steel and HWB. The experimental and the numerical results were used to propose suitable assumptions in order to develop an appropriate Strut-and-tie (STM) model for HSC beams with HWB and shear stirrups of span/depth ratio equals 3.

Introduction

High strength concrete is generally considered for a wide range of structural applications¹. The existing recommendations in AASHTO LRFD[2] code which has incorporated STM since 1994, ACI-318-08³ Appendix A, and EC2-04 [4] for use of STM to design shear are derived from research conducted essentially on Normal Strength Concrete (NSC) with cube strengths up to 50 MPa, and it was felt that these might not be applicable to High Strength Concrete (HSC) when a Horizontal Web Bar (HWB) is present in the beam.

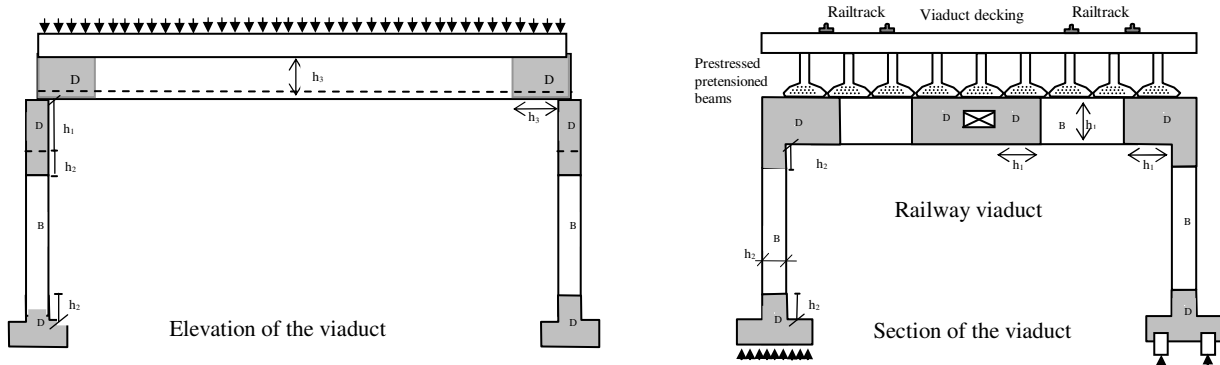


Figure 1: Structures are divided up into D-regions that extend the depth of the member each way from a reaction or discontinuity and B-regions, the parts of the structure between D-regions. The D-regions are isolated and the stresses on their boundaries are resolved into a number of individual forces acting on the boundaries of a D-region, as shown in Figure 6, Strut-and-tie models (STM) are then used to design the D-regions.

The present and recent [5] tests have shown that significant differences exist in the angle of crack of shear failure of NSC and HSC with HWB. In view of this, the current design recommendations of BS8110 [6] for the maximum allowable spacing of shear links were recommended [7] to be assessed in relation to HSC beams in shear. Previous investigations [8] have suggested that horizontal web steel can contribute to the overall shear resistance of a member in conjunction with other constituents, concrete, tension and shear steel. This research has covered an extensive experimental investigation on the contribution of HWB to shear resistance of HSC [9].

Experimental investigation

The size and the length of the test specimens were chosen to make the beams fail in shear ($a/d=3$) and to ensure that the specimens were sufficiently large to simulate real structural elements. Figure 2 shows the details of the eight beams which were 150×300mm in section and had a span of 2.2m. The HSC beams had two 6mm mild steel bars in the top only in the shear spans and the NSC beams had 2T20 in the top throughout.

For all beams the tension steel was 3T20 and shear links were R6 at 200mm centres in the shear spans. Both NSC and HSC beams were tested without and with horizontal web steel of 2T12, 2T20 and 2T25.

Tests were carried out on three specimens representing the steel in the links and the average value f_{yv} was 250 N/mm². The reinforcement used for the top, bottom and horizontal web steel was high yield, hot rolled deformed bars with a guaranteed yield value f_{yl} of 460 N/mm².

Table 1: All beams tested [9] are of shear span to depth (a/d) of 3.02 other than BJ-2 which has a/d=4.15

Beam No	Top Steel	Striup	Horizontal web steel	Cube Strength (f_{cu}) N/mm ²	Splitting strength (f_{sp}) N/mm ²	Ultimate load ($2V_u$) kN
NSC1	2T20	2R6-200	0	43.2	2.98	160
NSC2	2T20	2R6-200	2T12	41.0	3.01	203
NSC3	2T20	2R6-200	2T20	47.7	3.22	200
NSC4	2T20	2R6-200	2T25	43.3	2.97	210
HSC1-1	2R6	2R6-200	0	109.0	4.21	140
HSC1-2	2R6	2R6-200	0	101.2	-	143.3
HSC1-3	2R6	2R6-200	0	106.6	-	160.0
HSC2	2R6	2R6-200	2T12	109.3	5.20	265
HSC3	2R6	2R6-200	2T20	112.5	4.34	280
HSC4	2R6	2R6-200	2T25	112.5	4.34	300
NSCL	2T20	2R8-300	0	44.2	3.06	250
BJ-2	2T20	2R6-200	0	118.1	4.3	142

Details of concrete strengths, f_{cu} and f_{sp} are given in Table 1. In the concrete mix design, Rapid Hardening Portland cement was used in conjunction with 20mm gravel for NSC and 10mm limestone for HSC. f_{cu} was about 45 N/mm² for the NSC and 110 N/mm² for the HSC. For HSC the water: cement ratio was kept at 0.29 with the addition of admixtures. The beam specimens, the 150 mm (BS) cubes for NSC and 100mm BS cubes for HSC were cured in 28 days. The compressive strength tests were conducted on the same days as the beam tests. The concrete for all the beams was compacted using an immersion mechanical poker vibrator.

In beam test procedures, at each load increment, the vertical deflection at mid-span as well as the strains in the links, horizontal web bars and tensile reinforcing bars, were recorded. The development of cracks was also observed and recorded.

Test results and discussions

A summary of the test specimen details and results is given in table 1. The discussion of this part is presented in four sections (a) Shear failure loads; (b) Load-deflection behaviour; (c) Crack propagation (d) Load-strain behaviour:

(a) **Shear failure loads:** The first HSC1 failure load of 130 kN ($f_{cu} = 109$ N/mm²) appeared low, the second HSC1 failure load of 140 kN ($f_{cu} = 101.2$ N/mm²) and third failure load of 160 kN ($f_{cu} = 106.6$ N/mm²). The average ultimate load carried by these three similar HSC1 beams was 143.3 kN ($f_{cu} = 105.6$ N/mm²) as compared to ultimate load of beam NSC1 which was 160 kN ($f_{cu} = 43.2$ N/mm²). The links were similar in the two and neither contained any horizontal web steel. NSC1 did have 1.55% of compression reinforcement which was not present in HSC1. The inclination of the critical shear crack was much steeper in HSC1 at about 50° as compared with approximately 35° in NSC1.

The surprising reduction of shear resistance with increasing concrete strength found for beams NSC1 and HSC1 was reversed when horizontal web steel was provided. With two 25mm web bars in both, the ultimate loads for HSC4 ($f_{cu} = 112.5$ N/mm²) and NSC4 ($f_{cu} = 43.3$ N/mm²) were 300 kN and 210 kN respectively.

The major increase of shear strength for the HSC beams occurred between HSC1 (no horizontal web bars) and HSC2 (2T12) with ultimate loads of 130 kN and 265 kN. The rises with increasing horizontal web steel were much more modest - HSC3 (2T20) carried 280 kN and HSC4 (2T25) took 300kN.

With ordinary concrete the influence of horizontal bars was modest; NSC1 (no web bars)-160kN, NSC2 (2T12)-203kN, NSC3 (2T20)- 200kN and NSC4 (2T25)-210kN.

The results for the four high strength concrete beams with horizontal web steel demonstrated that no limit to improvement in shear resistance as the result of increasing the area of horizontal web reinforcement was reached. When the diameter of the web bars was increased from 20 to 25mm a further 7% improvement was recorded.

(b) **Load-deflection behaviour:** Mid-span deflections were measured by a single gauge mounted from the laboratory floor and include any settlements of the supports.

The deflection of beam HSC1 was fairly similar to that of NSC1. Both beams were without any horizontal web reinforcement the 1.55% of compression reinforcement, which was present in NSC1, reduced its deflection but the higher strength and elastic modulus of the concrete in HSC1 with no compression steel counter-weighted the compression steel in NSC1. The deflection of beam NSC1 was greater than for NSC4 (2T25) at equal loads and NSC1's deflection near failure was the greater.

The deflections of HSC2, HSC3 and HSC4 did not change by more than 15% as the area of horizontal web steel was increased in beams of high strength concrete.

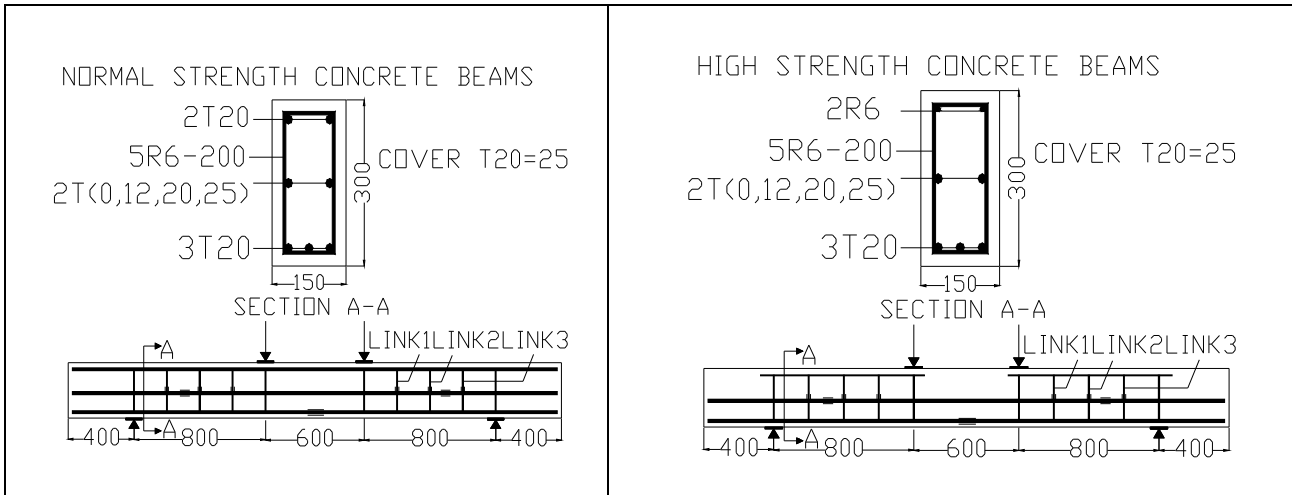


Figure 2: Geometry, Reinforcement details and position of strain gauges for all the Normal and High Strength Concrete for all test specimens other than NSCL and BJ-2 which have similar geometry for beams but their detailing is shown in Table 1. Horizontal Web Bars (HWB) are located at half the depth and are 2T (12,20 &25).

(c) Crack propagation: At loads of 40 to 60 kN, small flexural cracks appeared, at the bottom surface in the region of constant bending moment. As the load was increased new flexural cracks appeared in the shear spans spreading from the load application sections towards the supports and the flexural cracks in the shear spans tended to become somewhat inclined. This was followed by the sudden occurrence of a wide shear crack in one of the shear spans, which lead to failure.

A crack angle was defined as the angle between a tangent to the crack at the centre of the depth of the beam and its longitudinal axis.

The angle of the failure crack for the high strength concrete beam HSC1 was about 50° compared to the 35° for the normal strength concrete beam NSC1.

Beams HSC2, HSC3 and HSC4 had respective angles of cracks of about 43°, 45° and 42° compared to beams NSC2, NSC3 and NSC4 with angles of cracks 28°, 27° and 27°.

HSC1 and NSC2 had dowel cracks at the level of the bottom steel. These cracks were formed at 120kN (92% V_u) and 140kN (64% V_u). NSC3 and HSC4 may possibly have had dowel cracks in mid-web formed at 190kN (86% V_u) and 230kN (77% V_u). HSC3 and NSC4 developed web dowel cracks at 210kN (75% V_u) and 200kN (95% V_u).

(d) Load-strain behaviour: A comparison can be made between strains in links for the beams HSC4 and NSC4. Both beams had 2T25 horizontal web reinforcement

In the beam NSC4 links 1,2 and 3 yielded at 200 kN. Whereas, in HSC4 links 2 and 3 yielded at 200 kN and link 1 yielded at about 230kN. This shows that the difference between HSC and NSC is relatively small at the stage of stirrup yielding compared to the greater difference in failure load.

Beam HSC4 continued to sustain load for an increment of 100 kN after links 2&3 yielded and an increment of 70 kN after link 1 yielded. The horizontal web reinforcement (2T25) of HSC4 yielded at 270 kN.

One possible explanation is that the horizontal web reinforcement in beam HSC4 was stabilising arching. This resulted in yielding of the links and increased the forces in the main steel near supports. This tie effect of the tension steel continued until the tension reinforcement reached 90% of its yield strain at 300 kN when the beam failed.

The difference between high and normal strength concrete beams is partly in terms of the loads at which stirrups yielded. This difference could amount to a maximum load difference of 70 kN.

In beam HSC1 link 2 yielded at about 100 kN and link 3 reached 80% of its yield at 110kN. Shear failure occurred with a crack positioned between links 2 and 3. When failure occurred link 1 had not yet reached 40% of its yield, and the strain at mid-span of the tension steel had reached only 40% of its yield.

Proposal of an alternative design rule⁹

The shear resistance of rectangular reinforced concrete beams with vertical stirrups can be assessed by the BS8110 equation, which with safety factors eliminated, becomes;

$$V_{Rk,c} = 0.27 f_{cu}^{\frac{1}{3}} \rho^{\frac{1}{3}} \left(\frac{d}{400} \right)^{\frac{1}{4}} .bd \leq f_{cu}^{\frac{1}{2}} .bd \quad \text{for concrete only} \quad (1)$$

$$V_{Rk} = 0.27 f_{cu}^{\frac{1}{3}} \cdot \rho^{\frac{1}{3}} \cdot \left(\frac{d}{400} \right)^{\frac{1}{4}} \cdot bd + A_{sv} \cdot d \cdot \frac{f_{yv}}{s} \leq f_{cu}^{\frac{1}{2}} \cdot bd \quad \text{for concrete and stirrup} \quad (2)$$

In the code upper limits of $\rho < 3\%$ and $f_{cu} < 40 \text{ N/mm}^2$ are imposed. One way of assessing the total shear resistance of a member with a single layer of horizontal web steel is to add it's dowel resistance to the above V_{cu} . Using Baumann's¹⁰ dowel cracking expression:

$$D_{cr} = K \cdot b_n \cdot d_b \cdot f_{cu}^{\frac{1}{3}} \quad (3)$$

Baumann's equation is based on the idea that;

D_{cr} = Tensile strength of the concrete \times Net breadth of beam \times Primary bearing length

The primary bearing length is proportional to $= \sqrt[4]{(\text{flexural stiffness of dowel})/(\text{modulus of support})}$

When there are n dowel bars then

Flexural stiffness of total dowel = $n \times$ Stiffness of one bar.

The modulus of support ought to be practically independent of the number of bars. This suggests a change of Baumann's equation from

$$D_{cr} = K \cdot b_n \cdot d_b \cdot f_{cu}^{\frac{1}{3}} \quad \text{to} \quad D_{cr} = K \cdot b_n \cdot d_b \cdot \sqrt[4]{n} \cdot f_{cu}^{\frac{1}{3}} \quad (4)$$

To check if the movements of cracks should be sufficient for the mobilisation of D_{cr} , reference was made to published measurements of vertical movements at flexural cracks that developed into shear cracks. It was clear that the movements are large enough for dowel resistance to be fully achieved as it is limited by the tensile strength of the concrete, and a movement of about 0.1 mm can adequately mobilise it.

Hence if D_{cr} is adequately mobilised, the suggested formulation for the shear strength of the beam with stirrups and horizontal web reinforcement is;

$$V_{cu} = 0.27 f_{cu}^{\frac{1}{3}} \cdot \rho^{\frac{1}{3}} \cdot \left(\frac{d}{400} \right)^{\frac{1}{4}} \cdot bd + A_{sv} \cdot d \cdot \frac{f_{yv}}{s} + b_n \cdot d_b \cdot \sqrt[4]{n} \cdot f_{cu}^{\frac{1}{3}} \quad (5)$$

All the beams other than NSCL had 6 mm diameter single links at 200 mm centres.

$$\text{Therefore} \quad V_{lu} = A_{sv} \cdot d \cdot \frac{f_{yv}}{s} \quad (6)$$

where $A_{sv} = 56.6 \text{ mm}^2$, $f_{yv} = 250 \text{ N/mm}^2$, $d = 270 \text{ mm}$ & $s = 200 \text{ mm}$
Hence $V_{lu} = 19.1 \text{ kN}$, $100 A_s/bd = 2.33$, $d = 270$, $b = 150$,

From the modified Baumann equation

$$V_{bu} = 1.64 b_n d_b \sqrt[4]{n} \cdot f_{cu}^{\frac{1}{3}} \quad (7)$$

$$= 1.95 b_n d_b f_{cu}^{\frac{1}{3}} \quad (\text{where } n = 2) \quad (8)$$

Table 2: Experimental values of ultimate shear resistance compared to values predicted consider average value for HSC1 for the beams HSC1-1, HSC1-2 and HSC1-3. NSCL and BJ-2 are excluded in this table.

N.B: BS 8110's limit on f_{cu} has been ignored

Beam No	NSC1	NSC2	NSC3	NSC4	HSC1	HSC2	HSC3	HSC4
f_{cu} (N/mm ²)	43.2	41.0	47.7	43.3	109.0	109.3	112.5	112.5
V_{cu} (kN)	56.1	55.2	58.0	56.1	76.4	76.5	77.2	77.2
V_{lu} (kN)	19.1	19.1	19.1	19.1	19.1	19.1	19.1	19.1
Web Steel	-	2T12	2T20	2T25	-	2T12	2T20	2T25
V_{bu} (kN)	-	10.2	15.6	17.1	-	14.1	20.7	23.5
V_{calc} (kN)	91	100	108	108	111	125	132	135
V_{test} (kN)	80	101.5	100	105	65	132.5	140	150
V_{test}/V_{calc}	0.88	1.02	0.93	0.97	0.59	1.06	1.06	1.11

Shear weakness in HSC

A group of tests shown in table 3, gives an indication of a potential problem with high strength limestone aggregate concrete. In considering these results, it should be observed that the amount of shear reinforcement used in the HSC beams was below the minima of both EC 2 [4] and the Concrete Society [11] recommendations, which are $\rho_w f_y \geq 0.08$ and $\rho_w f_y \geq 0.039 f_{cu}^{2/3}$. Nonetheless, it is quite striking that the ratio of the ultimate shear to the characteristic resistance, calculated by the BS equation without a limit on f_{cu} and ignoring the requirement on $\rho_w f_y$, was as low as 0.69 with beam HSC1.

This is of special interest to the Iranian engineering community because most aggregate source near Tehran are limestone.

Table 3: Ratio of empirical values of ultimate shear resistance compared to predicted value from BS8110 for beams without horizontal web bars.

All beams have $\rho_w f_y b d = 34.5 \text{ kN}$ other than NSCL which has $\rho_w f_y b d = 31.4 \text{ kN}$

	Top bars $\rho'1\%$	Shear span a/d	Concrete Strength $F_c \text{ N/mm}^2$	Test failure $V_u \text{ kN}$	BS8110 concrete $V_{rk} \text{ kN}$	Ratio stirrups V_u/V_{rk}	Ratio concrete $V_u/V_{rk,c}$
Beam No.							
NSC1	1.58	3.02	34.6	80	51.6	1.08	1.44
NSCL	1.58	3.02	38.6	125	88.0	1.42	2.21
HSC1	0.14	3.02	94.0	65	71.9	0.69	0.86
HSC1-2	0.14	3.02	86.2	70	69.9	0.76	0.95
HSC1-3	0.14	3.02	91.6	80	71.3	0.85	1.06
BJ-2	1.58	4.15	103.1	71	74.2	0.74	0.90

The ultimate strengths of three of the four HSC beams were below both that of a reference beam with gravel aggregate and a modest value f_{cu} and the resistances calculated ignoring the shear steel.

In current UK recommendations, BS 8110 imposes a limit of 40 N/mm^2 on the value of f_{cu} to be used in its expression. The Concrete Society's recommendations of 1998 [11] had a limit of 100 N/mm^2 , but this has been reduced to 60 N/mm^2 , by an amendment made in 2004, motivated by this research at the University of Westminster¹².

Figure 3: Comparison of accuracy of BS8110 design rule to experimental values from Table 3

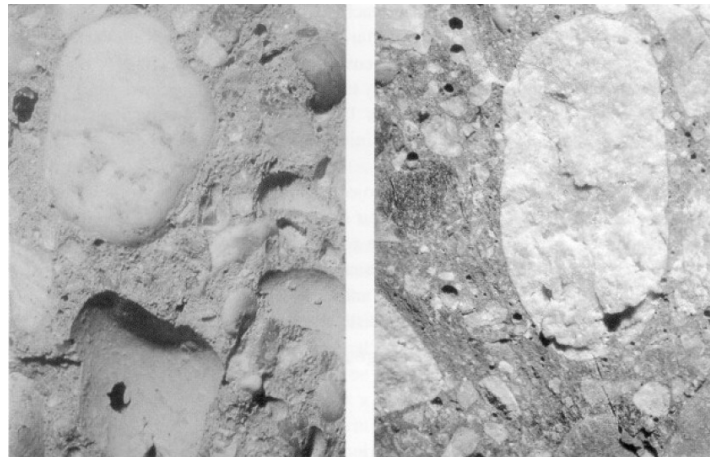
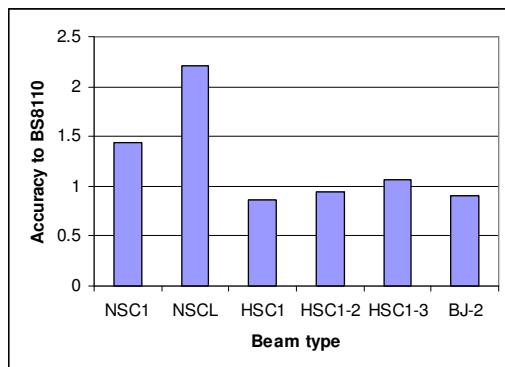


Figure 4: Aggregate interlock in NSC (left) & HSC (right)

Aggregate interlock is the result of the roughness of cracks in concrete. This roughness occurs when the aggregate in the concrete is stronger than the cement matrix as is the case with NSC. However, when the cement matrix is strong as in HSC, aggregate interlock becomes relatively a less significant factor.

In NSC where cracks travel around the unbroken aggregates that are clamped in the cement matrix, the aggregates produce a very rough surface that can transfer shear stresses. Displacement along the shear interface develops this 'friction'.

Crack friction depends on the aggregate strength and size as well as the difference of strength between the aggregate and the cement matrix. When cracks travel through the aggregates which normally occurs in HSC or in light weight NSC the roughness of the concrete is reduced and this reduces the crack friction capacity. The relationship of frictional resistance to concrete strength was rather uncertain until it was proved that this frictional resistance reduces in HSC (Figure 4).

Test results indicate that shear friction in HSC can be as low as 35% of that of NSC. This was demonstrated Walraven, J.C. in his paper "Shear Friction in High Strength Concrete," published in Progress in Concrete Research, Delft University of Technology in 1995.

Proposal for strut and tie model

The improved shear performance of HSC beams with stirrups and HWB of $a/d=3.02$ mainly due to dowel forces from the HWB are assumed to bear on wedges of concrete which, the action from these wedges are assumed to be transmitted into the main internal structural system to produce compression struts at 45° . This additional internal structural system within the length of $0.5z$ can be assumed to increase the strut and tie action from the conventional $a/z=2.5$ to 3.

HSC3 beam (see table 1) with HWB and stirrups of $a/d \leq 3.02$ was numerically modelled by F.E with detail stress trajectories were produced for beam HSC3 (see figure 5).

From following the path of the concentration of stresses the strut and tie model was developed and solved for the internal forces inside the shear span of the beam.

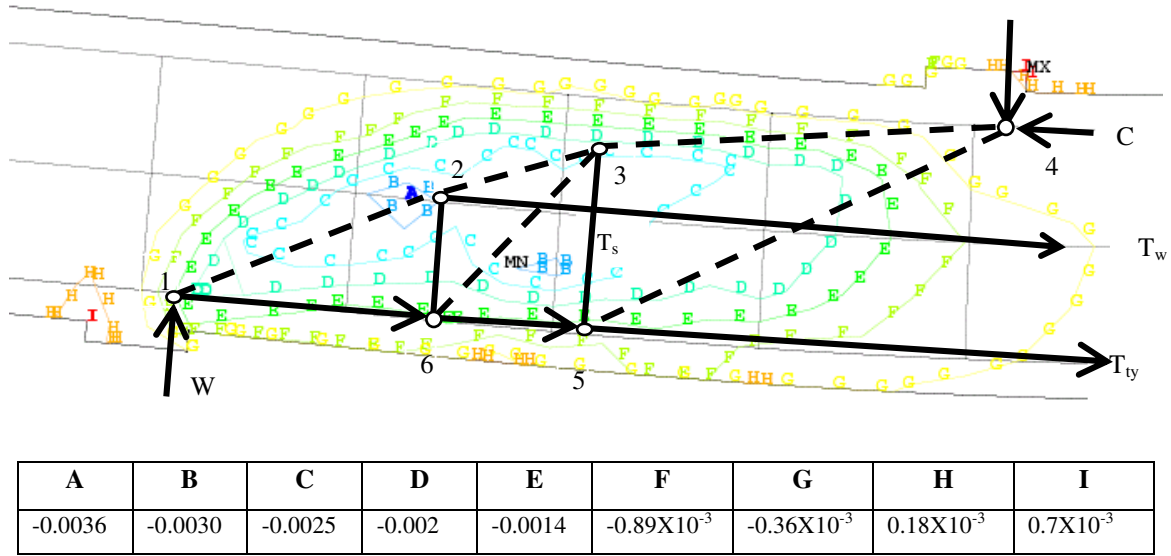


Figure 5: Position of the strut and tie model in relation to the location of shear stress trajectories from FE analysis of beam HSC3

Computing the strut and node dimensions for the STM

Lines A to I are the stress trajectories from F.E analysis of the experimental beam HSC3. The lines A to I in the figure represent the flow of compression stresses. Struts 1-2, 2-3 and 3- 4 replace the line of the maximum compression C on the upper trajectories and strut 4-5 on the lower trajectory (see figure 5). Strut 3-6 is due to dowel forces crossing the point of highest compression MN. Diagonal strut 3-6 acts as a secondary compression member to improve the overall performance of the truss. This part of the truss enclosed within nodes 2,3,5 and 6 is the outcome of the dowel action produced from the HWBs, this region extend the discontinuous loading and supporting regions to join one another to act as single D region.

For node 1 and prismatic compression strut 1-2, side elevation of which is shown in figure 6, the same bearing pressure is considered for each side of the node. This is a hydrostatic nodal zone because the in-plane stresses in the node are the same in all directions.

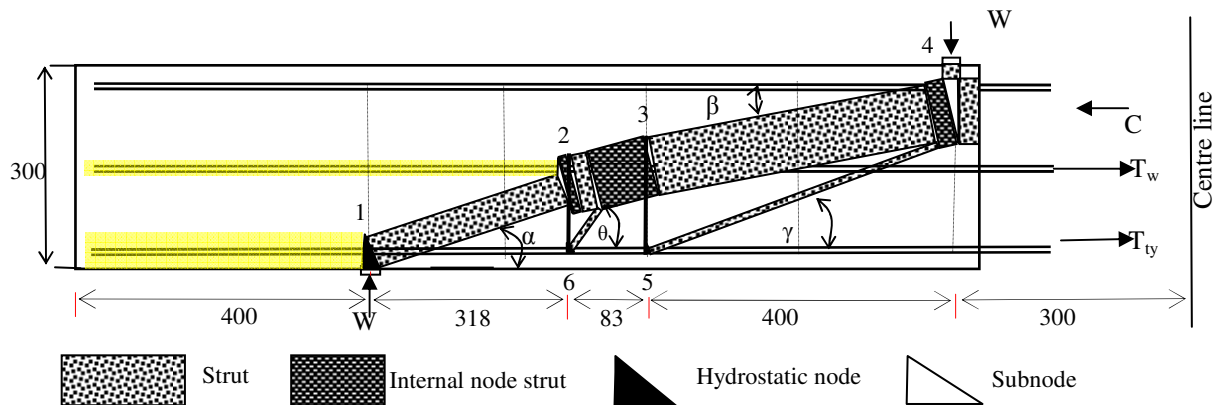


Figure 6: Strut-and-tie model for HSC beam with HWB of $a/d=3.02$ with stirrup. The sum of the shear transmitted is counter balanced by vertical components of the deflected strut which has an improved arching action as well as by the stirrups.

Table 2: Calculation of the forces in the Strut- and-tie model of beam HSC3 to ACI-318-08 Appendix A

Member	Vertical component force (kN)	Horizontal component force (kN)	Axial force (kN)	Effective concrete strength f_{ce} (MPa)	Minimum width of strut or nodal zone w_s (mm)
Node 1	135.0	0.0	135.0	61.2	19.6
1-2	135.0	378.0	401.0	61.2	58.2
$T_{s(1-6)}$	0.0	378.0	377.6	57.4	58.5
2-3	114.0	450.0	464.4	61.2	67.5
$T_{d(2-6)}$	21.0	0.0	20.6	57.3	3.2
T_w	0.0	72.0	72.5	57.4	11.2
3-6	21.0	11.0	23.8	61.2	3.5
3-4	109.0	461.0	473.6	61.2	68.8
$T_{s(3-5)}$	26.0	0.0	26.1	57.4	4.1
4-5	26.0	45.0	52.4	61.2	7.6
Node 4	135.0	0.0	135.0	76.5	15.7
Node 4	0.0	506.0	506.0	76.5	58.8
$T_{s(5-6)}$	0.0	388.0	387.9	57.4	60.1
T_{tv}	0.0	433.0	433.5	57.4	67.2
INS 2	149.0	445.6	469.9	61.2	68.3
INS 3	134.8	461.7	481.0	61.2	69.9
INS 4	135.1	506.3	524.0	61.2	76.1

Table 3: The angle each strut makes to horizontal

Strut	1-2	2-3	3-4	4-5	3-6
Horizontal angle	α	φ	β	γ	θ
Degrees	19.67°	14.25°	13.26°	29.90°	60.16°

The node dimensions can be decided in relation to the width of the support a strut-and- tie model. As we have a tensile force from within tension reinforcement, the width of that side of the node 1 is calculated from a hypothetical bearing plate on the end of the tie, which is assumed to exert a bearing pressure on the node equal to the compressive stress in the strut at that node. This is a C-C-T joint because this node is compressed in two directions and is anchoring a tie in one direction where strain incompatibility resulting from tensile steel strain adjacent to the compressive concrete strain reduces the strength of the nodal zone, therefore a reduction factor to Appendix A of ACI318-05 β_n will apply.

Nodes 2, 3 and 4 have 4 forces acting on the node which need to be simplified by introduction of internal strut as shown in figure 6.

Conclusion

- The use of strain gauges, a Demec enabled the cracking and deformation of slender reinforced high strength and normal strength concrete beams with stirrups, with and without horizontal web steel to be investigated up to peak load.
- Design rules proposed as the result of previous research by S. B. Desai [8] hold fair for the beams tested here. His rules produce reasonable estimates of ultimate shear resistance.
- Design rules proposed by BS8110 for normal strength concrete beams, with stirrups, and without horizontal web reinforcement are not valid if extrapolated to high strength concrete beams.
- Research by Desai, and the present tests on normal strength concrete beams with stirrups shows that for normal strength concrete, there is a limit to the maximum contribution of a central bar for beams with or without links.
- In general the tests on high strength concrete beams proved that horizontal web reinforcement located towards the centre of the beam improves the shear resistance significantly.
- The ultimate strength of three of the four HSC beams with limestone aggregate were below that of a reference beam with gravel aggregate and a modest value f_{cu} and the resistances calculated ignoring the shear steel.
- The results for beams HSC1 compared with HSC2, HSC3 and HSC4 showed an enhancement of shear resistance of about 130% when horizontal web steel is provided.
- Research by Desai [8] shows that the horizontal bars can provide, for design purposes, when considering fire exposure, their location protected by the surrounding concrete would be of some advantage.
- Further research will be required to find more realistic design factors for the improvement of the STM for shear design of high strength reinforced concrete members with horizontal web bar.

Notation

A_{st}	is the amount of tension steel (mm^2)
A_{sv}	is the area of cross-section of a link (mm^2)
A_b	is the area of cross-section of horizontal web steel (mm^2)
a	shear span from the centre of a concentrated load to the centre of a support
b	is the width of the cross-section of a beam (mm)
b_n	is the net breadth of the beam at level of dowels reinforcement (mm)
d	is the effective depth of the cross-section (mm)
d_b	is the diameter of horizontal web bar (mm)
f_{cu}	is the mean cube strength of concrete (N/mm^2)
f_{yl}	is the yield for longitudinal reinforcement (N/mm^2)
f_{yv}	is the yield strength for stirrups reinforcement (N/mm^2)
f_c	cylinder compression strength of concrete
f_{cu}	cube compression strength of concrete
f_y	yield stress of reinforcement
s	is the spacing of links along the length of the member (mm)
V_{bu}	is the contribution of central bars to V_u (kN)
V_{calc}	is the calculated ultimate shear strength (kN)
V_{cu}	is the contribution of concrete to V_u (kN)
V_{lu}	is the contribution of links to V_u (kN)
V_{test}	is the measured ultimate shear strength (kN)
V_u	is the ultimate shear resistance of a section (kN)
$V_{Rd,c}$	calculated design shear resistance of a member without shear reinforcement
V_{Rk}	calculated characteristic shear resistance
$V_{Rk,c}$	calculated characteristic shear resistance of a member without shear reinforcement
$\rho = 100 A_{st}/bd$	
$\rho_b = 100 A_b/bd$	
ρ_l	ratio of tension reinforcement (A_s/bd)
ρ'_l	ratio of compression reinforcement (A'_s/bd)
ρ_w	ratio of web reinforcement (vertical stirrups)

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